**Peer Graded Assignment**

1. Say whether the following is true or false and support your answer by a proof.



**Answer:**

It is true.

Since we are looking for existence of m and n, and for m=4 and n=0, the statement holds true.

(3\*4 + 5\*0 =12)

2. Say whether the following is true or false and support your answer by a proof: The sum of any five consecutive integers is divisible by 5 (without remainder).

**Answer:**

It is true.

Proof:

Suppose the integers are n, n+1, n+2, n+3, n+4. Sum of these five terms is equal to 5n +10 = 5(n+2).

So the sum of any five consecutive integers will be equal to the starting integer plus 2 and whole multiplied by 5. Since the sum is multiple of 5, So it is divisible by 5.

3. Say whether the following is true or false and support your answer by a proof: For any integer n, the number n2 + n + 1 is odd.

**Answer:**

It is true.

Proof:

Here we have two cases (a) n is odd, (b) n is even.

Case (a) n is odd;

If n is odd then n2 is also odd ……….. multiplying odd numbers results in a odd number, axiom of number theory.

If n is odd then n2 + n is an even number, because addition of odd numbers containing even number of terms results in an even number, axiom of number theory.

n2 + n + 1 will be always an odd number because n2 + n is even and adding 1 to this will result an odd number.

Case (b) n is even;

If n is even then n2 is also even ……….. multiplying even numbers results in even number, axiom of number theory.

If n is even then n2 + n is an even number, because addition of even numbers results in an even number, axiom of number theory.

n2 + n + 1 will be always an odd number because n2 + n is even and adding 1 to this will result an odd number.

4. Prove that every odd natural number is of one of the forms 4n+1 or 4n+3, where n is an integer.

**Answer:**

It is true.

Proof;

By division algorithm we know that each division operation results in following.

Numerator/Denominator= Quotient, Remainder……. (N/D=Q,R).

This essentially means N=D\*Q + R

Now let m is an odd natural number, we divide it by 4.

m/4= n, R which can be written as m=4n+R

while dividing any natural number, possible remainders are 0,1,2, 3.

But we know that m is an odd number, and 4 multiplied by n always result is an even number, so subtracting an even number i.e. 4n from an odd number i.e. m must result an odd remainder which in this case can be 1 and 3. (Note that essentially we are partitioning natural numbers into modulo 4 classes).

So, m=4n+1 OR m=4n+3

5. Prove that for any integer n, at least one of the integers n; n + 2; n + 4 is divisible by 3.

**Answer:**

It is true.

Proof;

By division algorithm, all integers can be classified in following modulo 3 classes

3k, 3k+1, and 3k+2 for some k.

If n is of the form 3k, then it is clearly divisible by 3.

If n+2 then consider the case 3k+1, which means n+2=3k+1+2=3k+3 =3(k+1) so divisible by 3.

If n+4 then consider the case 3k+2, which means n+4=3k+2+4=3k+6 =3(k+2) so divisible by 3.

So, at least one of the integers n; n + 2; n + 4 is divisible by 3.

6. A classic unsolved problem in number theory asks if there are infinitely many pairs of `twin primes', pairs of primes separated by 2, such as 3 and 5, 11 and 13, or 71 and 73. Prove that the only prime triple (i.e. three primes, each 2 from the next) is 3, 5, 7.

**Answer:**

It is true.

Proof;

Proof;

By division algorithm, all integers can be classified in following modulo 3 classes

3k, 3k+1, and 3k+2 for some k.

Let n , n+2, n+4 be sequence of odd natural numbers with n>3

If n is of the form 3k, then it is clearly divisible by 3.

If n+2 then consider the case 3k+1, which means n+2=3k+1+2=3k+3 =3(k+1) so divisible by 3.

If n+4 then consider the case 3k+2, which means n+4=3k+2+4=3k+6 =3(k+2) so divisible by 3.

So, at least one of the integers n; n + 2; n + 4 is divisible by 3.

Hence there cannot be prime triple except 3, 5, 7

7. Prove that for any natural number n,

2 + 22 + 23 +….. + 2n = 2n+1 - 2

**Answer:**

Let M=2 + 22 + 23 + : : : + 2n

Then 2M= 22 + 23 + : : : + 2n+2n+1

Now we subtract M from 2M which gives 2M-M=2n+1 – 2

Since 2M-M =M , so it holds identity.

Hence, 2 + 22 + 23 + ….. + 2n = 2n+1 - 2

8. Prove (from the definition of a limit of a sequence) that if the sequence  tends to limit L as n →∞, then for any fixed number M > 0, the sequence tends to the limit ML.

**\*Answer:**



9. Given an infinite collection An; n = 1; 2; : : : of intervals of the real line, their intersection is defined to be



Give an example of a family of intervals An; n = 1, 2…., such that An+1  An for all n and



Prove that your example has the stated property.

**\*Answer:**



10. Give an example of a family of intervals An; n = 1; 2; : : :, such that An+1  An for all n and  consists of a single real number. Prove that your example has the stated property.

**\*Answer:**



\*Here it is acknowledged that for question no. 8/9/10, I adopted solution from course material.